

Measurements of $\sin 2\beta$ in B decays

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I describe the experimental determination of the CKM angle β from measurements of the decay time separation Δt between B^0 mesons produced as $B\bar{B}$ pairs in e^+e^- annihilation. The current results from leading and non-leading decay modes are presented and compared.

1 Introduction

A primary objective of particle physics since the 1964 discovery of CP non-conservation in weak decays has been the understanding of the effect as a natural feature of the structure of the weak interaction. A clue was offered in 1973 by Kobayashi and Maskawa in anticipation of the discovery of a third fermion family. With three families the unitary transformation to the weak isospin basis of the left-handed fermions has four parameters, including one complex phase that breaks the CP symmetry of the flavor-changing transitions. In the Wolfenstein parameterization the CKM matrix V is

$$V = \begin{pmatrix} V_{ud} = 1 - \frac{1}{2}\lambda^2 & V_{us} = \lambda & V_{ub} = A\lambda^3(\rho - i\eta) \\ V_{cd} = -\lambda & V_{cs} = 1 - \frac{1}{2}\lambda^2 & V_{cb} = A\lambda^2 \\ V_{td} = A\lambda^3(1 - \rho - i\eta) & V_{ts} = -A\lambda^2 & V_{tb} = 1 \end{pmatrix}, \quad (1)$$

where $\lambda \simeq \sin \theta_c \simeq 0.22$ and $A \sim 1$. The unitarity equations, such as $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$, lead to “unitarity triangles” in which the angle of interest here is

$$\beta = \phi_1 = \arg(-V_{cd}V_{cb}^*/V_{td}V_{tb}^*). \quad (2)$$

2 Time evolution of the decay and CP violation

For a neutral B meson that is in the state B^0 at $t = 0$ we write the time-dependent amplitude for its decay to final state f as

$$\langle f | H | B_{\text{phys}}^0(t) \rangle = e^{-imt} e^{-\Gamma t/2} \left[A_f \cos \frac{1}{2} \Delta m t + i \frac{q}{p} \bar{A}_f \sin \frac{1}{2} \Delta m t \right], \quad (3)$$

where $A_f \equiv \langle f|H|B^0 \rangle$ is the amplitude for flavor-definite B^0 decay to the final state f , \bar{A}_f is the corresponding amplitude for \bar{B}^0 , and p, q give the weak eigenstates $B_{L,H}^0$ in the (B^0, \bar{B}^0) basis: $|B_{L,H}^0 \rangle = p|B^0 \rangle \pm q|\bar{B}^0 \rangle$. The average and difference between H and L masses are m and Δm , respectively, and we've taken the approximation $\Gamma_H = \Gamma_L = \Gamma$ for the decay rates. Particle-antiparticle mixing is responsible for the non-exponential behavior. When f is accessible to both B^0 and \bar{B}^0 the violation of CP symmetry appears through the interference between mixing (q/p) and decay (\bar{A}_f/A_f), even if CP is conserved in both ($|q/p| = |\bar{A}_f|/|A_f| = 1$). In $\Upsilon(4S)$ decay a $B^0\bar{B}^0$ pair is created in a $C = -1$ eigenstate, and the two mesons oscillate coherently between B^0 and \bar{B}^0 until one decays (Einstein-Podolsky-Rosen effect).

In the experiments there is no marker of $t = 0$; rather one must consider the time separation Δt between the decay of one B to a flavor eigenstate ("tag"), and the decay of the other B to the CP eigenstate f . The resulting decay rate in terms of Δt is

$$\frac{d\Gamma_{\pm}^f(\Delta t)}{d\Delta t} \propto e^{-|\Delta t|/\tau} (1 \pm \mathcal{I}m\lambda_f \sin \Delta m\Delta t), \quad (4)$$

where $\lambda_f \equiv \frac{q}{p} \frac{\bar{A}_f}{A_f}$, we've assumed $|\lambda_f| = 1$, and the $+$ ($-$) sign labels a B^0 (\bar{B}^0) tag. The ability to measure Δt depends on the motion of the $\Upsilon(4S)$ in the rest frame of the experiment, leading to direct measurement of $\Delta z \simeq \beta\gamma c\Delta t$ (z being the boost axis). The boost magnitudes for the two asymmetric B factories are $\beta\gamma = 0.56$ for PEP-II, and $\beta\gamma = 0.425$ for KEKB.

In the analysis one reconstructs the decay of one B meson in the final state f , assumed here to be a CP -eigenstate. Coming from $\Upsilon(4S) \rightarrow B^0\bar{B}^0$ (or B^+B^-), the B meson is nearly at rest ($p_B^* \simeq 325$ MeV/ c). This leads to a strong correlation between the reconstructed mass and the missing mass of the partner B . The usual choice of an independent pair of kinematic variables is

$$\Delta E = E_B^* - E_{\text{beam}}^*, \quad m_{\text{ES}} = \sqrt{E_{\text{beam}}^{*2} - |\mathbf{p}_B^*|^2}. \quad (5)$$

Here the asterisk denotes $\Upsilon(4S)$ frame, and the subscript B denotes the reconstructed B . One looks for central values of these variables peaked at $\Delta E = 0$ and $m_{\text{ES}} = m_B$. Typical resolutions are $\sigma(m_{\text{ES}}) \simeq 3$ MeV/ c^2 and $\sigma(\Delta E) \simeq 15 - 50$ MeV

The other (tag) B is not fully reconstructed; we need to know only its decay point and whether it's B^0 or \bar{B}^0 . The particles left over from reconstruction of the $B \rightarrow f$ candidate are therefore examined to form the recoil B vertex and to deduce its flavor. The resolution on Δz , of which the largest contribution comes from tag side, is $\simeq 180$ μm , or ~ 1.25 ps, and is similar for *BABAR* and *Belle*.

Flavor tagging signatures include the sign of charge in $(B^0 \rightarrow \ell^+, \bar{B}^0 \rightarrow \ell^-)$, $(B^0 \rightarrow K^+, \bar{B}^0 \rightarrow K^-)$, leading charged track, etc. The efficiency ϵ and mistag rate w for each algorithm is measured with reconstructed flavor eigenstate decays. The effective efficiency is given by $Q = \epsilon(1 - 2w)^2$. The average values of this figure of merit are $(28.1 \pm 0.7)\%$ reported by *BABAR*, and $(28.8 \pm 0.6)\%$ by *Belle*.

After convolution with the resolution function \mathcal{R} and provision for imperfect tagging the formula for the decay rate becomes

$$\frac{1}{\Gamma^f} \frac{d\Gamma_{\pm}^f(\Delta t)}{d\Delta t} = \frac{e^{-|\Delta t|/\tau}}{4\tau} [1 \pm (1 - 2w)\mathcal{I}m\lambda_f \sin \Delta m\Delta t] \otimes \mathcal{R}. \quad (6)$$

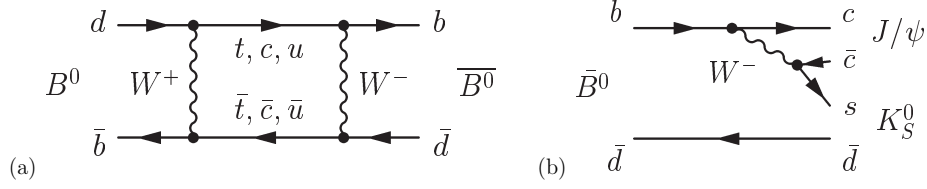


Figure 1: (a) $B\bar{B}$ mixing diagram; (b) leading diagram for $B \rightarrow \text{charmonium } K_S^0$ decays.

Thus we are looking for an asymmetry between B^0 - and \bar{B}^0 -tagged events having a sinusoidal Δt dependence with known angular frequency Δm and amplitude given by $\mathcal{I}m\lambda_f$ multiplied by the known dilution factor $1 - 2w$. The physics is in $\mathcal{I}m\lambda_f$; it contains the factor q/p , common to all decay modes, that can be calculated from the diagram in Fig. 1a. With the heavy b quark we have confidence in the short-distance calculation at parton level. The virtual t quark dominates in the loop, since its large mass is responsible for violating the GIM mechanism that otherwise suppresses the mixing. The result is

$$\frac{q}{p} = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}, \quad (7)$$

which in the Wolfenstein phase convention is $e^{-2i\beta}$.

3 $\sin 2\beta$ from charmonium $K^{0(*)}$ modes

The charmonium $K^{0(*)}$ decays are well described in terms of the color-suppressed, CKM-favored tree diagram of Fig. 1b. The ratio of amplitudes entering into λ_f is given by

$$\frac{\bar{A}_f}{A_f} = \eta_f \left(\frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}} \right) \left(\frac{p}{q} \right)_K = \eta_f \left(\frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}} \right) \left(\frac{V_{cd}^* V_{cs}}{V_{cd} V_{cs}^*} \right) = \eta_f, \quad (8)$$

where $\eta_f = +1$ (-1) for a CP even (odd) final state f , and the last step assumes the Wolfenstein phase convention. Combining with Eq. 7 we find, independent of phase convention,

$$\lambda_f = \eta_f e^{-2i\beta}, \quad \mathcal{I}m\lambda_f = -\eta_f \sin 2\beta. \quad (9)$$

Thus the amplitude of the sine term in Eq. 6 gives directly $\sin 2\beta$.

BABAR [1] and *Belle* [2] have performed this measurement with samples combining several charmonium K_S^0 modes, as well as $J/\psi K_L^0$ (for which $\eta_f = +1$). The contributing modes are listed in Fig. 2 which conveys a sense of the high purity of these samples. The exposures correspond to 88 (85) million produced $B\bar{B}$ pairs for *BABAR* (*Belle*). The Δt distributions from both experiments are shown in Fig. 3. From a 34-parameter likelihood fit to 2641 tagged events (with 78% purity), *BABAR* measure $\sin 2\beta$ given in Eq. 10. The corresponding *Belle* measurement, from 2958 tagged events (having 81% purity), is given in Eq. 11.

$$(\text{with } |\lambda_f| = 1) \sin 2\beta = 0.741 \pm 0.067 \pm 0.034 \quad \text{BABAR} \quad (10)$$

$$= 0.719 \pm 0.074 \pm 0.035 \quad \text{Belle} \quad (11)$$

These results agree well with each other, and together impose a significant new constraint on the upper vertex of the CKM triangle, consistent with prior knowledge.

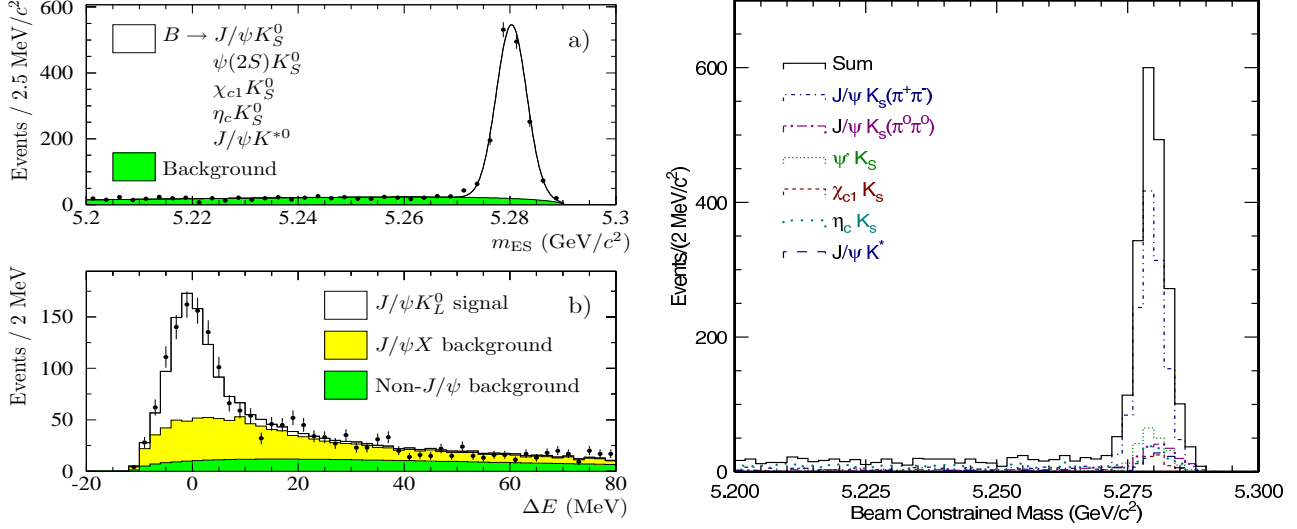


Figure 2: Distributions in kinematic variables for selected events with flavor tags from *BABAR* (left) and *Belle* (right). For the K_S^0 modes (CP odd, right and upper left) the m_{ES} distribution is shown. For K_L^0 (CP even, lower left), the energy residual is given.

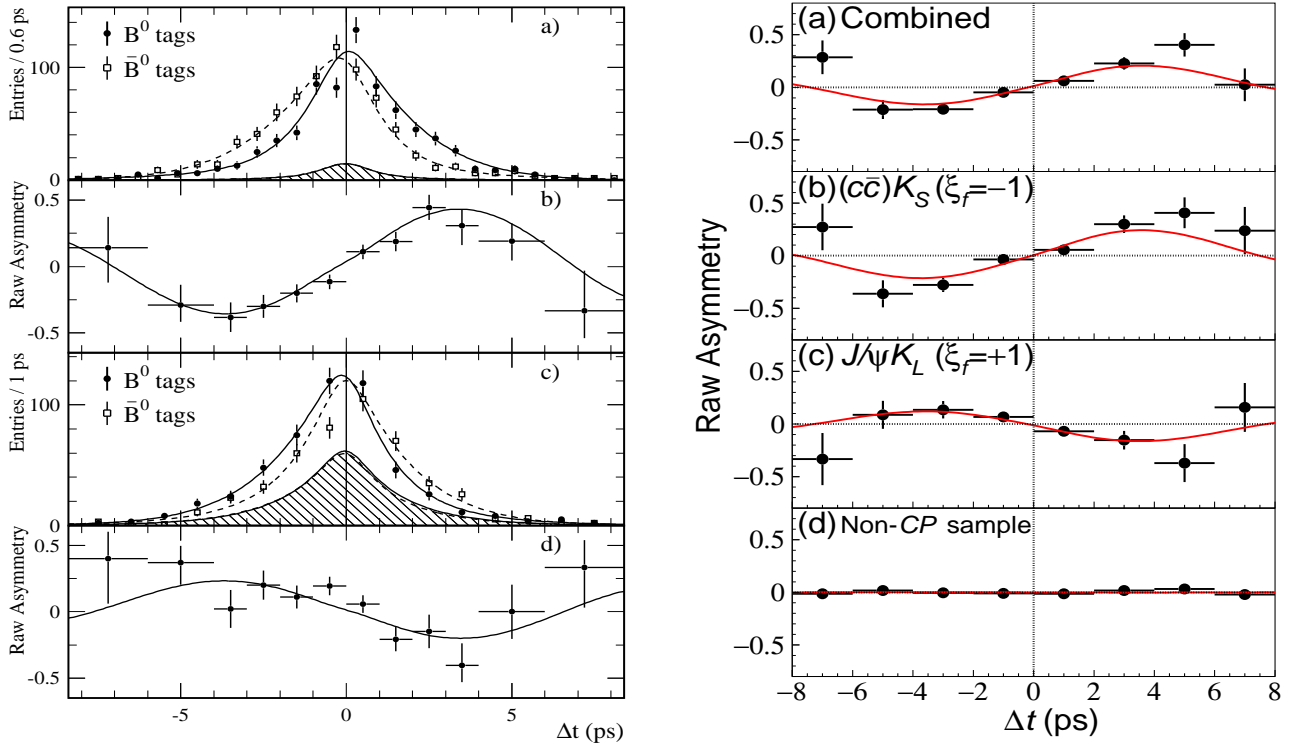


Figure 3: Rate and flavor asymmetry vs Δt from *BABAR* (left) and *Belle* (right). On the left appear (a, c) the tagged-sample rates and (b, d) asymmetries for (a, b) CP -odd and (c, d) CP -even modes. On the right the asymmetries are given for (a) all modes, for (b) CP -odd ($\xi_f = \eta_f = -1$) and (c) CP -even modes, and for (d) a flavor-definite control sample.

4 $\sin 2\beta$ and probes for new physics with rare B decays

Violations of CP symmetry can be also observed in a number of rarer B decays. In contrast with the $\mathcal{O}(\lambda^2)$ $b \rightarrow c\bar{c}s$ modes we've been discussing, these have couplings of $\mathcal{O}(\lambda^3)$, or are expected to be suppressed because their amplitudes contain penguin loops, or both. We include among these the decays with Cabibbo-suppressed $b \rightarrow c\bar{c}d$ (e.g., charmonium π^0 , open charm pair) and the gluonic penguin $b \rightarrow s\bar{q}q$ (e.g., ϕK_S^0 , $\eta' K_S^0$). These processes are sensitive to the presence of possible new physics, because with their smaller amplitudes interference terms are relatively more prominent, and because of possible virtual particles (e.g., SUSY) in penguin loops. These experiments are harder because of lower rates, higher backgrounds, and complications in their interpretation including the simultaneous presence of tree and penguin amplitudes, multiple penguin amplitudes, uncertainties from long-distance effects, etc.

In the time-evolution formalism for these decays we account for effective direct and interference CP -violating contributions by removing the assumption $|\lambda_f| = 1$ and introducing coefficients S_f and C_f of the sine- and cosine-like terms: for a CP eigenstate f we have

$$\frac{1}{\Gamma_f} \frac{d\Gamma_{\pm}^f(\Delta t)}{d\Delta t} = \frac{e^{-|\Delta t|/\tau}}{4\tau} \{1 \mp \Delta w \pm (1 - 2\langle w \rangle) [S_f \sin(\Delta m_d \Delta t) - C_f \cos(\Delta m_d \Delta t)]\} \otimes \mathcal{R}, \quad (12)$$

where

$$S_f \equiv \frac{2\mathcal{I}m\lambda_f}{1 + |\lambda_f|^2}, \quad -\mathcal{A}_f = C_f \equiv \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}, \quad (13)$$

$\Delta w = w(B^0) - w(\bar{B}^0)$, and as in section 2 we assume $\Gamma_H = \Gamma_L$. For reference, $S_f = \sin 2\beta$ and $C_f = 0$ (no direct CP violation in the decay) for $B^0 \rightarrow J/\psi K_S^0$.

4.1 $b \rightarrow c\bar{c}d$ decays

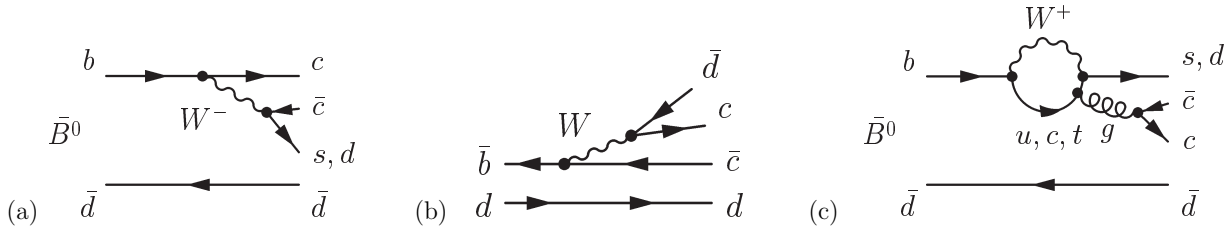


Figure 4: Diagrams for $b \rightarrow c\bar{c}q$ decays; (a) color-suppressed tree; (b) external tree; (c) gluonic penguin.

In section 3 we considered only tree (T) decay amplitudes. In general we have a competition between amplitudes like those shown in Fig. 4. There are two independent terms among the penguins (P) with (u, c, t) in the loop, and for the interpretation here we care about whether these bring in weak phases different than that of the tree amplitude. The non-GIM-suppressed pieces are $P_c - P_t$, which has the same CP phase as T , and $P_u - P_t$, which has a different CP phase. For $b \rightarrow c\bar{c}s$, such as $J/\psi K_S^0$, the ratio of $P_u - P_t$ to T is $\mathcal{O}(\lambda^4/\lambda^2)$, justifying our neglect of it. But for Cabibbo-suppressed $b \rightarrow c\bar{c}d$ decays such as $J/\psi \pi^0$, both are $\mathcal{O}(\lambda^3)$, and thus the P/T ratio becomes a theoretical systematic for the interpretation of $b \rightarrow c\bar{c}d$ decays.

With this caveat, we quote the results of measurements in the $b \rightarrow c\bar{c}d$ counterpart $B^0 \rightarrow J/\psi\pi^0$ of the charmonium- K_s^0 decays: *BABAR* [3], with 40 ± 7 signal events from $88 \times 10^6 B\bar{B}$ pairs find

$$S_{J/\psi\pi^0} = 0.05 \pm 0.49 \pm 0.16 \quad C_{J/\psi\pi^0} = 0.38 \pm 0.41 \pm 0.09, \quad (14)$$

and Belle [4], with 57 total events (86% purity) from $85 \times 10^6 B\bar{B}$ measure

$$S_{J/\psi\pi^0} = 0.93 \pm 0.49 \pm 0.11^{+0.27}_{-0.03} \quad -C_{J/\psi\pi^0} = \mathcal{A} = -0.25 \pm 0.39 \pm 0.06. \quad (15)$$

Both measurements are preliminary. They are consistent with $S_{J/\psi\pi^0} = \sin 2\beta$, $C_{J/\psi\pi^0} = 0$, within large errors.

BABAR have measured the time evolution for two modes with open charm pairs. Again the quark-level process is $b \rightarrow c\bar{c}d$ (and so $P_u - P_t$ again brings in a second weak phase), although there is in this case no color-suppression of the tree (see Fig. 4b); the estimate is $\Delta\beta \sim 0.1$, where $\Delta\beta$ is the deviation of the measured β_{eff} from true β .

The $D^{*\pm}D^\mp$ decays are not CP eigenstates, but are accessible from B^0 and \bar{B}^0 . The decay chains analyzed are $D^{*\pm} \rightarrow \pi^\pm D^0$ with four D^0 modes, and $D^+ \rightarrow K\pi\pi, K_s^0\pi$. With 113 ± 13 signal events from $88 \times 10^6 B\bar{B}$ pairs the results are [5]

$$S_{-+} = -0.24 \pm 0.69 \pm 0.12 \quad C_{-+} = -0.22 \pm 0.37 \pm 0.10 \quad (16)$$

$$S_{+-} = -0.82 \pm 0.75 \pm 0.14 \quad C_{+-} = -0.47 \pm 0.40 \pm 0.12. \quad (17)$$

Here S_{+-} corresponds to $D^{*+}D^-$, etc., and if one assumes equal amplitudes for $D^{*-}D^+$ and $D^{*+}D^-$ one expects $C_{-+} = C_{+-} = 0$; if penguins are negligible $S_{-+} = S_{+-} = -\sin 2\beta = -0.7$.

The same quark-level amplitudes describe $B^0 \rightarrow D^{*\pm}D^{*\mp}$. This, however, is a vector-vector decay, with odd- CP P -wave and even- CP S - and D -wave contributions. The final state is reconstructed in the chains $D^{*\pm} \rightarrow D^0\pi^\pm, D^\pm\pi^0$, excluding $B^0 \rightarrow D^+D^-\pi^0\pi^0$. An angular analysis [6] yields for the CP -odd fraction $R_\perp = 0.063 \pm 0.055 \pm 0.009$, i.e., this final state is $\sim 94\%$ CP -even. With 156 ± 14 signal events (before tagging, 73% purity) from $88 \times 10^6 B\bar{B}$ pairs, *BABAR* find the preliminary results

$$|\lambda_{f+}| = 0.75 \pm 0.19 \pm 0.02, \quad \mathcal{I}m\lambda_{f+} = 0.05 \pm 0.29 \pm 0.10, \quad (18)$$

where λ_{f+} refers to the CP -even component. We may compare this with the tree-level expectation $|\lambda_{f+}| = 1$ and $\mathcal{I}m\lambda_{f+} = -\sin 2\beta$.

4.2 $b \rightarrow s\bar{q}q$ decays

For $b \rightarrow s\bar{q}q$ decays with no c -quarks in the final state, the tree is a CKM-suppressed $b \rightarrow u$ with both color and Cabibbo suppression at the internal vertex. Therefore the leading amplitudes are gluonic $b \rightarrow s$ penguins. The ratio T/P is $\mathcal{O}(\lambda^4/\lambda^2)$. For $B^0 \rightarrow \eta'K_s^0$ the internal gluon converts to either an $s\bar{s}$ or $d\bar{d}$ pair, and these may interfere. The rather large (60×10^{-6}) branching fraction for this decay may in fact be the result of constructive interference of these amplitudes. In the case of ϕK_s^0 we have no tree, and only the penguins with $g \rightarrow s\bar{s}$. Estimates of $\Delta\beta$ from “tree pollution” are as small as 0.01 (0.025) for $B^0 \rightarrow \eta'K_s^0$ (ϕK_s^0) [7].

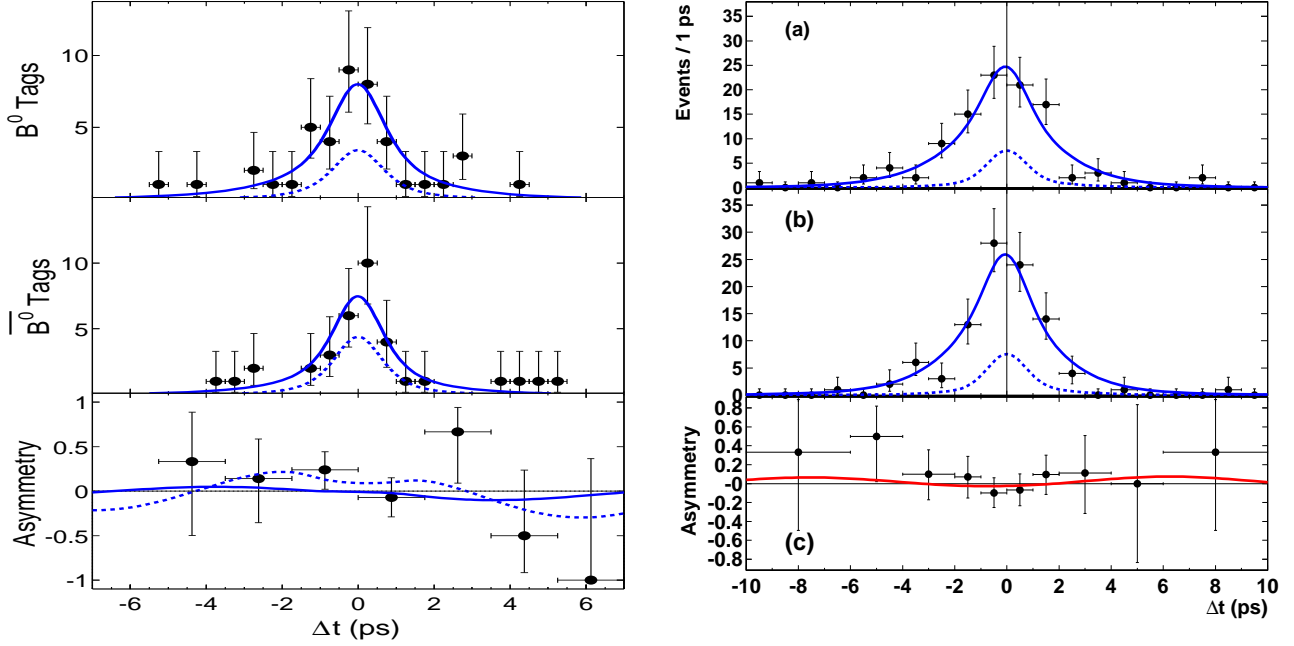


Figure 5: Tagged-sample Δt distributions and asymmetry for (left) $B^0 \rightarrow \phi K_S^0$ and (right) $B^0 \rightarrow \eta' K_S^0$ (BABAR, preliminary). Tags in the right-hand plot are (a) B^0 and (b) \bar{B}^0 . Dashed curves show background components of the best-fit functions.

For $B^0 \rightarrow \phi K_S^0$ the ϕ is reconstructed from its $K^+ K^-$ decay, and the K_S^0 from $\pi^+ \pi^-$ (BABAR include also $\pi^0 \pi^0$). With 51 signal events (31 tagged), from $87 \times 10^6 B\bar{B}$ pairs the BABAR preliminary results are [8]

$$S_{\phi K_S^0} = -0.18 \pm 0.51 \pm 0.07 \quad C_{\phi K_S^0} = -0.80 \pm 0.38 \pm 0.12, \quad (19)$$

($S_{\phi K_S^0} = -0.26 \pm 0.51$ when $C_{\phi K_S^0}$ is constrained to zero). Belle find with 53 total events (purity 67%) from $78 \times 10^6 B\bar{B}$ [9]

$$S_{\phi K_S^0} = -0.73 \pm 0.64 \pm 0.22 \quad \mathcal{A}_{\phi K_S^0} = -C_{\phi K_S^0} = -0.56 \pm 0.41 \pm 0.16. \quad (20)$$

The Δt distributions are shown in the left-hand plots in Figs. 5 and 6, respectively. Taken together these are somewhat at odds with the expectation $S_{\phi K_S^0} = \sin 2\beta = 0.7$; the errors are still large however.

From Belle there is also a measurement in non-resonant $B^0 \rightarrow K^+ K^- K_S^0$, a mixture of CP -odd and even states. Their analysis indicates that it is in fact about 97% even ($\xi_f = +1$), and with their 191 total events (purity 50%) they find (see center plot of Fig. 6) [9]

$$-\xi_f S = 0.49 \pm 0.43 \pm 0.11_{-0.00}^{+0.33} \quad \mathcal{A} = -C = -0.40 \pm 0.33 \pm 0.10_{-0.26}^{+0.00}. \quad (21)$$

The $B^0 \rightarrow \eta' K_S^0$ decay is reconstructed from $\eta' \rightarrow \eta \pi^+ \pi^-$ and $\eta' \rightarrow \rho^0 \gamma$. With Belle's sample of 299 total events (purity 49%) the measurements are (right-hand plot of Fig. 6) [9]

$$S_{\eta' K_S^0} = +0.71 \pm 0.37_{-0.06}^{+0.05} \quad \mathcal{A} = -C_{\eta' K_S^0} = +0.26 \pm 0.22 \pm 0.03 \quad (22)$$

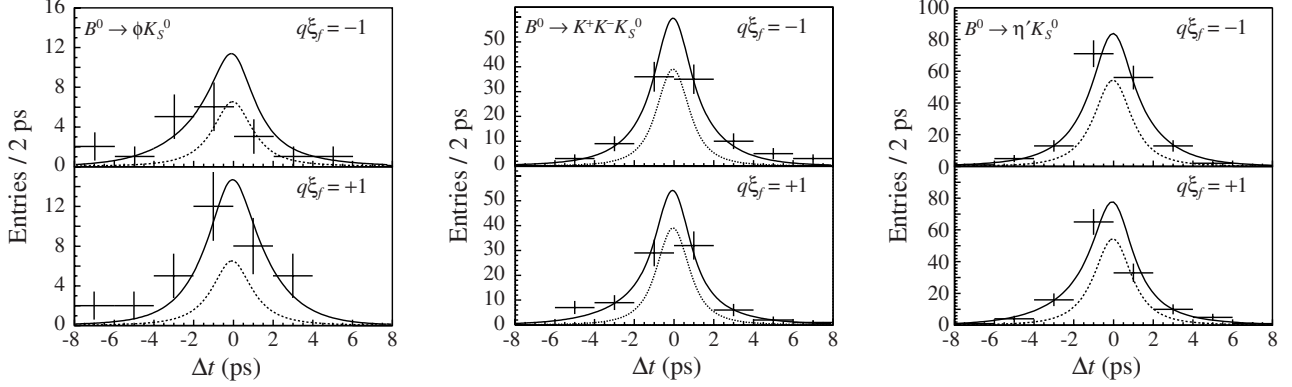


Figure 6: Tagged B^0 ($q = +1$) and \bar{B}^0 ($q = -1$) Δt distributions for $b \rightarrow sq\bar{q}$ decays (Belle).

(consistent with the charmonium value of $\sin 2\beta$), and from *BABAR*, with 109 tagged signal events (purity 70%) from 89×10^6 $B\bar{B}$ pairs (right-hand plot of Fig. 5) [10]

$$S_{\eta' K_S^0} = 0.02 \pm 0.34 \pm 0.03, \quad C_{\eta' K_S^0} = 0.10 \pm 0.22 \pm 0.03. \quad (23)$$

This (preliminary) null result is still consistent within errors with $\sin 2\beta = 0.7$.

5 Summary

All of the measurements presented here are summarized in Fig. 7 [11]. The shaded bands show results of averaging separately the measurements made with B decays to charmonium and with the charmless penguin-dominated modes. Future confirmation of the separation of the two bands would challenge the standard model, but considerable improvement in the theoretical understanding as well as in the experimental measurements would be needed for a definitive conclusion.

We have seen that CP non-conservation is well established in B^0 decays, and the effect is large in the interference between mixing and decay. The effect is well accommodated in the universal weak interactions of the quarks embodied in the standard CKM model. We are just beginning to explore further whether anything new is happening in processes where standard model effects are suppressed. We see hints of inconsistencies in β from charmless decays, where several channels are now being measured, with only more data needed for definitive results.

Acknowledgments

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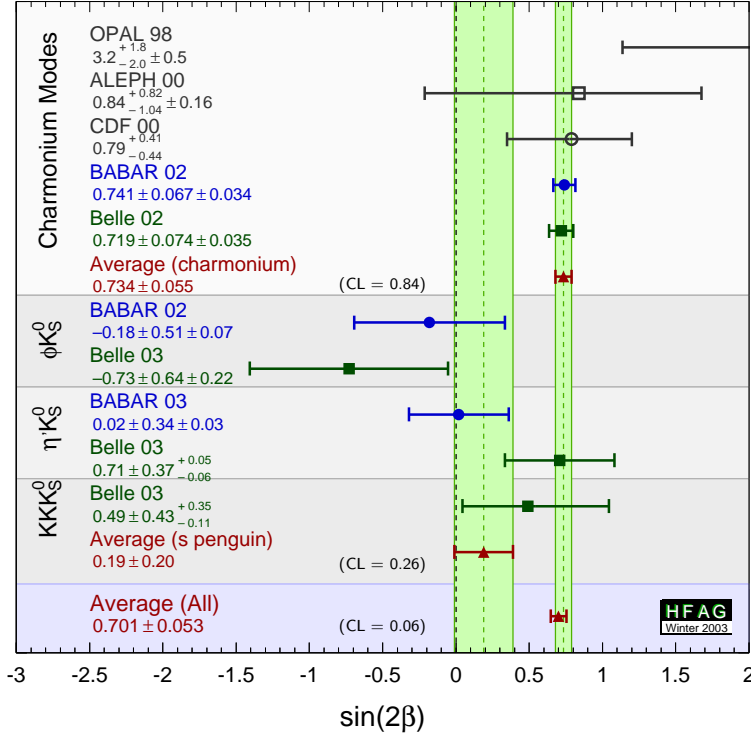


Figure 7: Measurements of $\sin 2\beta$ and their world averages.

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